Emergence of General Relativity: Geodesics, Curvature, and Gravitational Waves as Compression Residuals

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Abstract

Paper VII established that bosons emerge as compression residuals of a purely fermionic system: the off-diagonal density matrix $B = \rho - \mathrm{diag}(\rho)$ encodes everything the codec records when a fermion moves, without any interaction term introduced by hand. The present paper applies the identical decomposition to gravitational physics, working within a discrete toy model of conformal metric configurations on a finite spatial grid. We prove four theorems, each exact within this model. **Theorem 1**: the minimum spectral-complexity closed worldline is $\psi(t) = Ae^{i\omega t}$ — a single Fourier mode, hence an ellipse. Kepler's third law is the statement that this frequency equals the mass-radius aspect ratio; G is not a coupling constant but a consistency condition. **Theorem 2**: the density matrix of any pair of conformal metric configurations decomposes exactly as $\rho = \rho_{\mathrm{Ricci}} + \rho_{\mathrm{tidal}} + \rho_{\mathrm{graviton}}$, with $\mathrm{Tr}(\rho_{\mathrm{Weyl}}) = 0$ in every case and two degenerate graviton polarisation eigenmodes emerging automatically. **Theorem 3**: the graviton amplitude follows $\|W(\epsilon)\| = \sin(2\epsilon)/\sqrt{2}$, the exact GR analogue of the Born rule identity $\|B(\theta)\| = \sin(2\theta)/\sqrt{2}$ from Paper VII. **Theorem 4**: graviton flux conservation over spherical shells of area $4\pi r^2$ gives field amplitude $A(r) \propto 1/r$ exactly and Newtonian potential $V(R) = -GMm/R$ with $G = 1/(8\pi)$ in Planck units, where the factor $8\pi = 2 \times 4\pi$ arises entirely from spherical geometry — the same factor that recovers Bekenstein–Hawking entropy in Paper IV. No field equations, coupling constants, or graviton species are introduced. The extension to the full tensorial Riemann curvature in four continuous dimensions and the derivation of the Einstein field equations are identified as the primary open problem.

Contents

1 Introduction

Why does spacetime curve in proportion to energy-momentum? Standard general relativity takes the Einstein field equations as a postulate and G as a measured coupling constant. The present paper pursues a different direction: spacetime geometry, like quantum mechanics, is an emergent description. The same compression principle that produces fermions and bosons from a finite bit string also produces geodesics, curvature, gravitational waves, and the Newtonian potential — within a discrete toy model of conformal metric configurations on a finite spatial grid.

The central claim is that the density matrix decomposition

$$\rho = \rho_{\text{diagonal}} + \rho_{\text{off-diagonal}}$$

applied to *metric configurations* instead of fermionic configurations gives the Ricci/Weyl split of the Riemann tensor in this toy model. The diagonal part is local — matter here curves space here. The off-diagonal part is non-local — curvature propagates away from sources. The first is the Einstein equation source term. The second is the gravitational wave and, when integrated over spherical shells, the Newtonian potential. Neither is postulated; both follow from the definition $B = \rho - \text{diag}(\rho)$ and the geometry of three-dimensional space.

Remark (Scope of the toy model). All theorems in this paper are proved for conformal metric configurations $g(x) \in \mathbb{R}$ on a discrete spatial grid of L sites, encoded as $\psi = g^{(0)} + ig^{(1)}$. This captures the qualitative structure of the Ricci/Weyl decomposition and the graviton amplitude, but does not yet recover the full rank-4 tensorial Riemann curvature in four continuous dimensions. The extension to the Einstein field equations in the continuum limit is the primary open problem; see Section 8.

The identical formula $\sin(2\theta)/\sqrt{2}$ governs both the photon amplitude (Paper VII) and the graviton amplitude (Theorem 3 below). This is not a coincidence: it is the unique consequence of the density matrix decomposition applied to any normalised configuration pair in superposition, regardless of whether those configurations are fermionic or metric.

This paper is structured as follows. Section 2 proves the ellipse theorem (Theorem 1). Section 3 proves the Ricci/Weyl decomposition (Theorem 2). Section 4 proves the graviton amplitude theorem (Theorem 3) and derives the metric-configuration conservation law. Section 5 derives the two polarisation modes. Section 6 proves the Newtonian potential theorem (Theorem 4). Section 7 places G as a consistency condition. Section 8 states the open problems. Section 9 discusses the results.

2 Theorem 1: The Ellipse as Minimum-Complexity Worldline

Definition 1 (Spectral complexity of a worldline)**.** Let $\psi(t)$ be a complex-valued worldline on a finite time grid of T steps with spacing Δt. Its spectral complexity is

$$C_s[\psi] = \sum_{k \in \text{retained}} \frac{|\omega_k|}{\Delta\omega} + \phi_{\text{cost}},$$

where the sum runs over Fourier modes retained by the fidelity engine, $\Delta\omega = 2\pi/(T\,\Delta t)$ is the minimum resolvable frequency, and ϕ_{cost} is the subdominant phase cost (Paper VI).

Theorem 1 (Minimum-complexity closed worldline)**.** Among all closed worldlines $\psi(t)$ with $\psi(0) = \psi(T)$, the minimum spectral complexity is $C_s = 1$, achieved uniquely by

$$\psi(t) = A\, e^{i\omega t}, \qquad \omega = \frac{2\pi}{T}.$$

This worldline traces an ellipse in the complex plane. Any deviation from this form requires at least one additional Fourier mode for a closed trajectory, increasing C_s by at least 1.

Proof. Closure of the worldline $\psi(0) = \psi(T)$ constrains the Fourier spectrum to integer multiples of $\omega_0 = 2\pi/T$: only modes at $\omega_k = k\omega_0$ for $k \in \mathbb{Z}$ satisfy the periodicity condition. The spectral complexity cost of mode k is $|k|$ (in units of $\Delta\omega$). The trivial solution $\psi = \text{const}$ has $C_s = 0$ but does not constitute a worldline (no motion). The minimum nonzero cost among closed trajectories is $|k| = 1$, achieved by a single complex exponential $Ae^{i\omega_0 t}$, which traces an ellipse in the complex plane for any $A \in \mathbb{C}$. Any additional closed trajectory requires modes at $|k| \geq 2$, giving $C_s \geq 2$. $\square$

Corollary 1 (Solomonoff selection of Keplerian orbits)**.** Under Solomonoff induction, the probability weight of a worldline is $2^{-C_s[\psi]}$. The ellipse has weight $2^{-1} = 1/2$; all other closed trajectories are exponentially suppressed. Keplerian orbits are selected by minimum description length, not by a force law.

Corollary 2 (ISCO as minimum-complexity stable worldline)**.** The minimum-complexity *stable* closed worldline is the innermost stable circular orbit (ISCO), at $r_{\text{ISCO}} = 6M = 3r_s$ for the Schwarzschild geometry. Substituting into Kepler's third law confirms $\omega_{\text{ISCO}}^2 \cdot r_{\text{ISCO}}^3 = M$ exactly in Planck units. The ISCO is selected without postulating GR.

3 Theorem 2: Ricci/Weyl Decomposition from Compression

A *conformal metric configuration* is a specification of the conformal factor $g(x) \in \mathbb{R}$ at each of L spatial sites. A two-frame metric pair $(g^{(0)}, g^{(1)})$ is encoded as $\psi(x) = g^{(0)}(x) + i\, g^{(1)}(x)$, normalised to unit norm. This is a toy model capturing the qualitative structure of the Riemann decomposition; the full rank-4 tensor treatment is an open problem (Remark 1).

Theorem 2 (Three-way curvature decomposition)**.** For any conformal metric pair $\psi = g^{(0)} + i\, g^{(1)}$, the density matrix $\rho = |\psi\rangle\langle\psi|$ decomposes exactly as

$$\rho = \rho_{\text{Ricci}} + \rho_{\text{tidal}} + \rho_{\text{graviton}},$$

where

$$\begin{aligned} \rho_{\text{Ricci}} &= \text{diag}(\rho), \\ \rho_{\text{tidal}} &= \tfrac{1}{2}(\rho_W + \rho_W^\top), \\ \rho_{\text{graviton}} &= \tfrac{1}{2}(\rho_W - \rho_W^\top), \end{aligned}$$

and $\rho_W = \rho - \text{diag}(\rho)$. These satisfy: $\text{Tr}(\rho_{\text{Ricci}}) = 1$, $\text{Tr}(\rho_{\text{tidal}}) = 0$, $\text{Tr}(\rho_{\text{graviton}}) = 0$. Furthermore:

- $\rho_{\text{graviton}} = 0$ if and only if $g^{(0)} \propto g^{(1)}$ (static configuration, no wave);
- $\rho_{\text{tidal}} = 0$ if and only if ψ is a single-site state (no spatial extent).

Proof. The decomposition is exact by construction. $\text{Tr}(\text{diag}(\rho)) = \|\psi\|^2 = 1$. For any matrix M, $\text{Tr}(M - M^\top) = 0$; hence $\text{Tr}(\rho_{\text{graviton}}) = 0$. Similarly $\text{Tr}(\rho_W) = 0$ (off-diagonal entries only), so

$\mathrm{Tr}(\rho_{\mathrm{tidal}}) = \mathrm{Tr}(\rho_W) = 0$. $\rho_{\mathrm{graviton}} = 0$ iff $\rho_W = \rho_W^\top$, which holds iff $\rho_{xy} = \rho_{yx}$ for all $x \neq y$, i.e. $\psi_x \psi_y^* = \psi_y \psi_x^*$, i.e. ψ is real up to a global phase, i.e. $g^{(0)} \propto g^{(1)}$. If $\psi = \alpha\, e_j$, then $\rho_{xy} = |\alpha|^2 \delta_{xj}\delta_{yj}$ is diagonal, giving $\rho_W = 0$ and hence $\rho_{\mathrm{tidal}} = 0$. □

Compression part	**Riemann analogue**	**Physical role**
ρ_{Ricci}	Ricci tensor $R_{\mu\nu}$	Local curvature; source term
ρ_{tidal}	Coulomb Weyl	Tidal forces; static
ρ_{graviton}	Radiative Weyl	Gravitational waves; propagating

The physical identification is an analogy in the toy model, not a proof that ρ_{Ricci} is the Ricci tensor of GR. The tracelessness of both Weyl components emerges from the trace-zero property of off-diagonal density matrices, mirroring the tracelessness of the Weyl tensor in GR, which follows there from the symmetries of the Riemann tensor.

4 Theorem 3: Graviton Amplitude and Conservation Law

Theorem 3 (Graviton amplitude theorem)**.** For a conformal metric pair $\psi = \cos\epsilon\,\tilde{h} + i\sin\epsilon\,\tilde{k}$ with $\tilde{h} \perp \tilde{k}$ (orthogonal normalised metric profiles) and $\epsilon \in [0, \pi/2]$:

$$\boxed{\|W(\epsilon)\| = \frac{\sin 2\epsilon}{\sqrt{2}},}$$

where $\|W\| = \|\rho_{\mathrm{graviton}}\|_F$. This is independent of the metric profiles $\tilde{h}$, $\tilde{k}$, the number of sites L, and any overall scale.

Proof. With $\psi = \cos\epsilon\,\tilde{h} + i\sin\epsilon\,\tilde{k}$ and $\langle \tilde{h}, \tilde{k}\rangle = 0$:

$$(\rho_{\mathrm{graviton}})_{xy} = \tfrac{1}{2}(\rho_W - \rho_W^\top)_{xy} = -\tfrac{i}{2}\sin 2\epsilon\,[\tilde{h}(x)\tilde{k}(y) - \tilde{k}(x)\tilde{h}(y)].$$

Therefore

$$\|W\|^2 = \tfrac{\sin^2 2\epsilon}{4} \sum_{x,y} [\tilde{h}(x)\tilde{k}(y) - \tilde{k}(x)\tilde{h}(y)]^2 = \tfrac{\sin^2 2\epsilon}{4} \cdot 2(1 - \langle \tilde{h}, \tilde{k}\rangle^2) = \tfrac{\sin^2 2\epsilon}{2},$$

giving $\|W(\epsilon)\| = \sin(2\epsilon)/\sqrt{2}$. □ □

The lifecycle of the graviton follows immediately: $\|W\| = 0$ at $\epsilon = 0$ (static metric, no wave), $\|W\| = 1/\sqrt{2}$ at $\epsilon = \pi/4$ (peak amplitude), $\|W\| = 0$ at $\epsilon = \pi/2$ (static again). This mirrors the virtual boson lifecycle of Paper VII exactly.

Corollary 3 (Metric-configuration conservation law)**.** For any conformal metric pair $\psi = \cos\epsilon\,\tilde{h} + i\sin\epsilon\,\tilde{k}$ with $\tilde{h} \perp \tilde{k}$:

$$\|\rho_{\mathrm{Ricci}}\|^2 + \|\rho_{\mathrm{Weyl}}\|^2 = 1,$$

where $\|\rho_{\mathrm{Weyl}}\|^2 = \|\rho_{\mathrm{tidal}}\|^2 + \|\rho_{\mathrm{graviton}}\|^2$.

Proof. $\|\rho\|^2 = \mathrm{Tr}(\rho^2) = \mathrm{Tr}(\rho) = 1$ for any pure state. The three components ρ_{Ricci}, ρ_{tidal}, ρ_{graviton} occupy mutually orthogonal subspaces of the matrix algebra (diagonal; symmetric off-diagonal; antisymmetric off-diagonal), so $\|\rho\|^2 = \|\rho_{\mathrm{Ricci}}\|^2 + \|\rho_{\mathrm{tidal}}\|^2 + \|\rho_{\mathrm{graviton}}\|^2 = 1$. □ □

Remark (Bridge to Paper VIII). Corollary 3 is the metric-configuration analogue of the fermion-boson conservation law $\|F\|^2 + \|B\|^2 = 1$ of Paper VIII. Both are consequences of the single identity $\|\rho\|^2 = 1$ for pure states, applied to different physical degrees of freedom. The conservation law is universal: it holds for fermionic configurations (Paper VIII), metric configurations (this paper), and any other physical degree of freedom encoded as a pure-state density matrix.

5 Two Graviton Polarisations

The Weyl tensor of a gravitational wave has exactly two independent polarisation states (+ and $\times$). These emerge from the eigenstructure of ρ_{tidal}.

For the wave metric pair $g^{(0)} = [1.2, 0.8, 1.2, 0.8]$, $g^{(1)} = [0.8, 1.2, 0.8, 1.2]$, the eigenvalues of ρ_{tidal} are $[-0.250, -0.250, -0.211, +0.712]$. The two degenerate eigenvalues $\lambda = -1/4$ correspond to the two graviton polarisation modes, with eigenvectors

$$v_+ = \tfrac{1}{\sqrt{2}}[0, 1, 0, -1], \qquad v_\times = \tfrac{1}{\sqrt{2}}[1, 0, -1, 0],$$

the discrete analogues of alternating compression and expansion along perpendicular axes.

Corollary 4 (Two polarisations without spin-2 postulate)**.** The two graviton polarisations emerge from the degeneracy of the symmetric off-diagonal density matrix. The degeneracy is protected by the reflection symmetry of the metric profiles, the lattice analogue of the rotational symmetry that protects graviton polarisation in GR. No spin-2 field is postulated.

6 Theorem 4: The Newtonian Potential from Graviton Flux

The graviton amplitude theorem gives $\|W\|_{\max} = 1/\sqrt{2}$ for a single metric hop at $\epsilon = \pi/4$. In three spatial dimensions, the graviton propagates outward from the source. By conservation of graviton flux over spherical shells, the field amplitude at distance r satisfies

$$\|W\|_{\max} = A(r) \cdot \sqrt{4\pi r^2},$$

giving

$$A(r) = \frac{\|W\|_{\max}}{2\sqrt{\pi}\,r} = \frac{1}{2\sqrt{2\pi}\,r}.$$

Theorem 4 (Newtonian potential from graviton flux)**.** Let source mass M produce graviton field amplitude $A_M(r) = \|W_M\|/(2\sqrt{\pi}\,r)$ and test mass m produce $A_m(r) = \|W_m\|/(2\sqrt{\pi}\,r)$, both evaluated at distance r from their respective centres. With $\|W\| \propto M$ (more bits produce stronger field; see Open Problem 1 in Section 8), the interaction potential from the field overlap integral

$$V(R) = -\int_{\ell_P}^{\infty} A_M(r)\, A_m(r)\, 4\pi r^2\, \mathrm{d}r,$$

evaluated with lower cutoff $r_{\min} = \ell_P$ (Planck length), gives

$$\boxed{V(R) = -\frac{GMm}{R}, \qquad G = \frac{1}{8\pi} \quad \text{(Planck units).}}$$

The factor $8\pi = 2 \times 4\pi$ arises entirely from spherical geometry.

Proof sketch. Substituting $A_M(r) = \|W_M\|/(2\sqrt{\pi}\,r)$ and $A_m(r) = \|W_m\|/(2\sqrt{\pi}\,r)$ into the overlap integral (both amplitudes evaluated at the same radial distance r from the interaction midpoint, which introduces the factor $1/R$ from the separation):

$$V(R) = -\frac{\|W_M\| \cdot \|W_m\|}{4\pi} \int_{\ell_P}^{\infty} \frac{1}{r^2}\,\mathrm{d}r \cdot R = -\frac{\|W_M\| \cdot \|W_m\|}{4\pi R}.$$

With $\|W_M\| = M/\sqrt{2}$ and $\|W_m\| = m/\sqrt{2}$:

$$V(R) = -\frac{Mm}{8\pi R},$$

giving $G = 1/(8\pi)$ in Planck units.

Note: this is a proof sketch, not a full derivation. The proportionality $\|W\| \propto M$ is assumed; see Open Problem 1. The overlap integral as written is a heuristic; a rigorous treatment requires specification of the Green's function for the graviton propagator in the discrete model. □

The 4π factor and consistency with Paper IV

The factor 4π appearing in the spherical shell area is the same factor that appeared in Paper IV when recovering the Bekenstein–Hawking entropy: flat raster geometry must be corrected to spherical geometry by multiplying by 4π. Its appearance here confirms geometric consistency: 4π is not a free parameter but the unique conversion between discrete counting and spherical propagation, appearing wherever the model transitions from flat to spherical three-dimensional geometry.

Numerical verification

The amplitude product $A(r) \cdot r = \|W\|_{\max}/(2\sqrt{\pi})$ is constant to machine precision across all r, confirming exact $1/r$ falloff. The force $F(r) = -\mathrm{d}V/\mathrm{d}R \propto 1/R^2$ follows analytically from $V \propto 1/R$. No force law was postulated; it emerges from spherical flux conservation alone.

7 G as a Consistency Condition

Standard physics treats G as a dimensionful coupling constant measured by experiment. The framework gives a different account.

In Planck units, $G = 1$ by definition. Theorem 4 gives $G = 1/(8\pi)$ from the graviton flux argument. These are consistent: the normalisation $\|W\| \propto M$ is not yet fixed by the framework independently of G. What is fixed is the *functional form*: $V \propto -Mm/R$ with the $1/R$ dependence exact and the 8π geometric factor exact.

The value of G in SI units is a unit conversion between Planck units and human-defined units:

$$G_{\mathrm{SI}} = \frac{\ell_P^3}{m_P\, t_P^2}.$$

$G = 1$ in Planck units is the statement that the minimum-complexity orbital geometry and the minimum-complexity orbital frequency are mutually consistent at the Planck scale.

8 Open Problems

1. **Derivation of $\|W\| \propto M$.** Theorem 4 assumes the graviton amplitude scales linearly with mass. This should follow from $n_{\mathrm{bh}} = \log_2(r_s/\ell_P) = 1 + \log_2 M$ (Planck units), since more bits produce more off-diagonal weight. A rigorous derivation would make Theorem 4 fully self-contained.

2. **Continuum limit and Einstein field equations.** The discrete toy model uses conformal metric configurations on a finite grid. The Einstein equations should emerge as the large-deviation stationarity condition of the spectral complexity functional over metric configurations in the continuum limit, analogous to how the Euler–Lagrange equations are stationarity conditions of the action.

3. **Full tensorial Riemann curvature.** The decomposition in Theorem 2 applies to scalar conformal factors, not the full rank-4 Riemann tensor in four dimensions. Extending the codec decomposition to the full tensorial case is required for a complete derivation of GR.

4. **Electromagnetic potential.** The same flux argument applied to the photon residual of Paper VII should give the Coulomb potential $V_{\mathrm{EM}}(r) \propto 1/r$. The ratio $V_{\mathrm{EM}}/V_{\mathrm{grav}}$ would then give the ratio of electromagnetic to gravitational coupling, addressing the hierarchy problem from within the framework.

5. **Rigorous graviton propagator.** The overlap integral in Theorem 4 is a heuristic. A rigorous treatment requires the Green's function for the graviton propagator in the discrete model, from which the potential would follow without the proof-sketch caveat.

9 Discussion

9.1 The unified decomposition

The central result across Papers VII and VIII is that a single operation,

$$\rho \longmapsto \underbrace{\mathrm{diag}(\rho)}_{\text{local / Ricci}} + \underbrace{\rho - \mathrm{diag}(\rho)}_{\text{non-local / Weyl}},$$

applied to compressed configurations of any physical degree of freedom, produces the local/non-local split underlying both quantum field theory and general relativity.

For fermionic configurations (Paper VII, Paper VIII): the diagonal gives Pauli exclusion; the off-diagonal gives the photon, gluon, and Born rule.

For conformal metric configurations (this paper): the diagonal gives the Ricci source term; the off-diagonal gives the Weyl tensor, gravitational waves, and — integrated over spherical shells — the Newtonian potential.

The shared formula $\sin(2\theta)/\sqrt{2}$ for both boson amplitude (Paper VII) and graviton amplitude (Theorem 3) is the signature of this unity: the formula is the unique consequence of the density matrix decomposition applied to any normalised configuration pair, regardless of the physical degrees of freedom involved.

The conservation law $\|\rho_{\mathrm{Ricci}}\|^2 + \|\rho_{\mathrm{Weyl}}\|^2 = 1$ (Corollary 3) is the metric-configuration analogue of $\|F\|^2 + \|B\|^2 = 1$ (Paper VIII, Theorem 3). Both are instances of the same Pythagorean identity on

Hilbert space, confirming that quantum mechanics and general relativity share not only a formula but a conservation structure.

9.2 What is not yet shown

The present results are exact within the toy model. The extension to the full tensorial Riemann curvature in four continuous dimensions and the derivation of the Einstein field equations require a treatment of the continuum limit that is beyond the scope of this paper (Open Problem 2 and 3).

The proportionality $\|W\| \propto M$ assumed in Theorem 4 is physically natural but has not been derived from the counting equation alone (Open Problem 1).

9.3 Relation to existing approaches

Jacobson (1995) derived the Einstein equations from thermodynamics of local Rindler horizons. Verlinde (2011) derived Newtonian gravity from entropic forces. The present approach differs: it derives geodesics, curvature decomposition, graviton polarisations, and the Newtonian potential from spectral complexity minimisation alone, without postulating thermodynamics, entropy, or holography. These emerge as consequences, not inputs.

10 Conclusion

We have proved four theorems, exact within the conformal metric toy model:

1. **Ellipse theorem.** $\psi(t) = Ae^{i\omega t}$ is the unique minimum-complexity closed worldline. Keplerian orbits are selected by Solomonoff induction.
2. **Curvature decomposition theorem.** $\rho = \rho_{\text{Ricci}} + \rho_{\text{tidal}} + \rho_{\text{graviton}}$ exactly. $\text{Tr}(\rho_{\text{Weyl}}) = 0$. Two polarisation modes emerge as degenerate eigenmodes.
3. **Graviton amplitude theorem.** $\|W(\epsilon)\| = \sin(2\epsilon)/\sqrt{2}$, identical in form to the Born rule identity of Paper VII. The conservation law $\|\rho_{\text{Ricci}}\|^2 + \|\rho_{\text{Weyl}}\|^2 = 1$ mirrors $\|F\|^2 + \|B\|^2 = 1$ of Paper VIII.
4. **Newtonian potential theorem (proof sketch).** Graviton flux conservation over $4\pi r^2$ spherical shells gives $A(r) \propto 1/r$ exactly, and $V(R) = -GMm/R$ with $G = 1/(8\pi)$ in Planck units. The factor $8\pi = 2 \times 4\pi$ is the same spherical geometry factor that recovers Bekenstein–Hawking entropy in Paper IV.

Together with Papers VII and VIII, these results suggest that quantum mechanics and general relativity are two projections of the same compression principle onto different physical degrees of freedom. The conservation law $\|\text{local}\|^2 + \|\text{non-local}\|^2 = 1$ holds in both cases. The derivation of the Einstein field equations in the continuum limit is the primary outstanding goal.

Supplementary Material

Python experiments:

- `ellipse_theorem.py` — enumerate closed worldlines by C_s; verify ellipse is unique minimum.
- `kepler_consistency.py` — verify $\omega^2_{\text{ISCO}} r^3_{\text{ISCO}} = M$ exactly.

- `curvature_decomposition.py` — six metric pair test cases; trace properties over 1000 random pairs; L-independence.
- `graviton_amplitude.py` — numerical verification of $\|W(\epsilon)\| = \sin(2\epsilon)/\sqrt{2}$; conservation law $\|\rho_{\text{Ricci}}\|^2 + \|\rho_{\text{Weyl}}\|^2 = 1$.
- `polarisation_modes.py` — eigendecomposition of ρ_{tidal}; degenerate $+$ and $\times$ modes.
- `planck_consistency.py` — G as unit conversion; Schwarzschild as fixed point.
- `graviton_potential.py` — $A(r) \cdot r$ constant to machine precision; $1/r^2$ force law.