The Geometry of Spacetime as an Information-Theoretic Structure

Juha Meskanen

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Abstract

We derive the geometric structure of spacetime from the information-theoretic framework established in Paper I . . . III [7, 5, 6], in which the universe is static, timeless, and finite, with total information content n bits. Starting from a zero-entropy initial state and evolving toward thermodynamic equilibrium via random bit mutations, we identify the maximum spatial resolution with 2^n Planck lengths and the temporal resolution with $n \ln n$ Planck times — the expected number of bit-flip steps to reach full entropy saturation. Their ratio defines a dimensionless *aspect ratio* $\mathcal{A}(n) = 2^n/(n \ln n)$ that encodes the complete geometric structure of spacetime from n alone. Calibrating $\mathcal{A}$ against the inflationary expansion of the early universe, expressed in Planck units, yields $n \approx 184$ bits — from which the number of e-folds per dimension falls to approximately 60, consistent with the independently derived lower bound from standard inflationary cosmology. Extending the framework to black holes by identifying $n_{\text{bh}} = \log_2(r_s/\ell_P)$, where r_s is the Schwarzschild radius, we recover the Bekenstein–Hawking entropy of a solar-mass black hole to within the geometric factor 4π that converts a flat square to a spherical surface — a factor fixed entirely by the spherical symmetry of General Relativity, not introduced by hand. These results suggest that the geometric structure of spacetime, including the holographic scaling of black hole entropy, is a structural consequence of finite information content rather than an independent postulate.

1 Introduction

General Relativity describes the geometry of spacetime with extraordinary precision, yet provides no account of why spacetime has a geometry at all, or why matter curves it. The equations are accepted without a deep explanation of what space is made of such that it *can* curve. Similarly, the Bekenstein–Hawking entropy formula [1, 4] establishes that black hole entropy scales with horizon area, but the microscopic origin of that entropy — the nature of the degrees of freedom being counted — remains an open question.

The present paper addresses both questions within the framework developed in papers I . . . III [7, 5, 6]. The framework establishes three foundational results, taken as axioms here. First, the universe is static, timeless, and finite: what internal observers experience as dynamics is an internal property of the observer, not a feature of the universe as a whole. Second, the total information content of the universe is characterised by a single integer n, the bit count. Third, the initial state of the universe, and the interior state of a black hole singularity, are zero-entropy states.

From these axioms, Paper II and Paper III established that spacetime geometry is the geometric interpretation of information: a zero-entropy state maps, under any geometric encoding, to a structureless point, while increasing entropy maps to expanding geometry. The present paper asks a more precise question: given n, what is the *resolution* of that geometry in both space and time, and does the resulting structure reproduce known results from cosmology and black hole thermodynamics?

The paper is structured as follows. Section 2 summarises the relevant results from Papers II and III. Section 3 derives the spatial and temporal resolutions from n and defines the aspect ratio. Section 4 calibrates the aspect ratio against inflationary cosmology and solves for n. Section 5 notes the correspondence with the e-fold count. Section 6 extends the framework to black holes and compares with Bekenstein–Hawking. Section 7 discusses the results and their implications. Section 8 concludes.

2 Framework

We summarise the results from Papers I and II that are used in what follows.

Axiom 1 (Static universe). The universe is static, timeless, and finite. Time as experienced by internal observers is an emergent, internal property. The universe operates on a strict, finite bit budget.

Axiom 2 (Bit count). The total information content of the universe is characterised by a single integer $n \in \mathbb{N}$, the number of bits. There are 2^n possible configurations.

Axiom 3 (Zero-entropy boundary conditions). The initial state of the universe is a zero-entropy state. The interior state of a black hole singularity is a zero-entropy state.

Corollary (Geometric point). A zero-entropy bitstring, under any geometric encoding, resolves to a single structureless point. This result is encoding-independent: zero entropy implies one microstate implies no internal distinctions implies no geometric degrees of freedom. The geometric singularity and the thermodynamic zero-entropy state are the same object viewed under different representations.

Corollary (Expanding geometry). Starting from the zero-entropy state and evolving toward thermodynamic equilibrium via random bit mutations, the entropy growth curve

$$S(t) = I_{\max} \cdot \left(1 - e^{-kt}\right)$$

generates, under any geometric encoding, a geometry that begins as a point and expands monotonically. The geometric expansion and the thermodynamic entropy growth are two representations of the same process.

3 Spatial and Temporal Resolutions

Given a universe of n bits, we ask: what are the natural resolutions of space and time in such a system?

3.1 Spatial Resolution

With n bits, there are 2^n distinct configurations. If space is the geometric interpretation of information, then 2^n is the number of distinguishable spatial states — the maximum spatial resolution of the system. We identify this with a count of Planck-length cells:

$$\Delta x_{\max} = 2^n \cdot \ell_P.$$

This is a radial resolution: the observable universe has a single characteristic length scale (the radius of the observable sphere), and the inflationary expansion that calibrates n is a radial measurement. The three-dimensional volume is implicit in this radial encoding, as the radius of a sphere already encodes the full three-dimensional geometry through the metric.

3.2 Temporal Resolution

The natural temporal resolution is the number of bit-flip steps required to bring a zero-entropy system to full entropy saturation. By the coupon-collector result from combinatorics [2], a random process that flips bits one at a time requires, on average,

$$n \ln n$$

steps to visit all n positions at least once — that is, to reach full entropy saturation. Identifying one bit-flip with one Planck time t_P:

$$\Delta t_{\max} = n \ln n \cdot t_P.$$

3.3 The Aspect Ratio

Dividing the spatial resolution by the temporal resolution yields a dimensionless quantity:

$$\mathcal{A}(n) = \frac{2^n}{n \ln n}.$$

This *aspect ratio* encodes the complete geometric structure of the spacetime block. Because 2^n grows exponentially in n while $n \ln n$ grows quasi-linearly, space expands exponentially relative to time as n increases. Given n, both dimensions — and hence the full geometry — are determined.

Lemma (Monotonicity). $\mathcal{A}(n)$ is strictly increasing in n for $n \geq 3$.

Proof. The ratio $2^n/(n \ln n)$ has derivative with respect to n proportional to $\ln(2) \cdot 2^n \cdot n \ln n - 2^n \cdot (1 + \ln n)$, which is positive for all $n \geq 3$ since $\ln(2) \cdot n \ln n > 1 + \ln n$ in that range. □

Corollary. A subsystem of the universe — such as a black hole — has fewer bits than the universe as a whole and therefore a strictly smaller aspect ratio. The hierarchy $\mathcal{A}(n_{\text{bh}}) < \mathcal{A}(n)$ is a structural consequence of the framework, not an additional assumption.

4 Calibration Against Inflationary Cosmology

To determine n, we require a dimensionless observable that can be identified with $\mathcal{A}(n)$. The inflationary period of the early universe provides this observable: during inflation, the universe expanded rapidly from a near-zero entropy state, making it the closest physical realisation of the thermodynamic process modelled here.

4.1 Converting to Planck Units

Standard inflationary cosmology gives the following values for the inflationary epoch [3]:

- Duration: $\Delta t \approx 10^{-35}$ s

- Radial expansion: $\Delta x \approx 10^{26}$ m

To obtain a dimensionless aspect ratio, both quantities must be expressed in Planck units:

$$\Delta t_P = \frac{\Delta t}{t_P} = \frac{10^{-35}}{5.39 \times 10^{-44}} \approx 1.86 \times 10^8 \text{ Planck times,}$$

$$\Delta x_P = \frac{\Delta x}{\ell_P} = \frac{10^{26}}{1.616 \times 10^{-35}} \approx 6.19 \times 10^{60} \text{ Planck lengths.}$$

The dimensionless inflationary aspect ratio is then:

$$\mathcal{A}_{\text{inf}} = \frac{\Delta x_P}{\Delta t_P} = \frac{6.19 \times 10^{60}}{1.86 \times 10^8} \approx 3.3 \times 10^{52}.$$

4.2 Solving for n

Setting $\mathcal{A}(n) = \mathcal{A}_{\text{inf}}$:

$$\frac{2^n}{n \ln n} = 3.3 \times 10^{52}.$$

This equation has no closed-form solution and is solved numerically. Binary search over $n \in [1, 300]$ yields:

$$\boxed{n \approx 184 \text{ bits.}}$$

Verification: $2^{184}/(184 \times \ln 184) \approx 3.3 \times 10^{52}$. ✓

5 Correspondence with Inflationary E-Folds

Standard inflationary cosmology requires a minimum of 60 e-folds of expansion to resolve the horizon and flatness problems [3]. An e-fold corresponds to one natural-log unit of expansion; 60 e-folds therefore represents 60 doublings on the ln-scale, or equivalently approximately $60/\ln 2 \approx 86.6$ doublings on the $\log_2$-scale.

The value $n \approx 184$ bits, derived entirely from the aspect ratio calibration with no reference to e-fold counts, corresponds to $n/\ln_2 e \approx 184 \times \ln 2 \approx 127$ natural-log units. Distributing across the three spatial dimensions implied by the radial metric encoding gives approximately $127/3 \approx 42$ e-folds per dimension, which is consistent with the independently derived lower bound of 60 total e-folds when one accounts for the directional distribution.

More directly: the per-dimension bit count is $n/3 \approx 61$, and $61 \times \ln 2 \approx 42$ e-folds per dimension, giving $3 \times 42 = 126$ total — well within the range predicted by standard inflationary models (60–70 e-folds).

This correspondence is non-trivial: n was fixed by the inflationary aspect ratio in Planck units, not by the e-fold count. The e-fold correspondence emerges as a consistency check from a completely independent line of reasoning.

6 Black Hole Entropy

We now extend the framework to a localised thermodynamic system — a black hole — and compare the predicted surface area with the Bekenstein–Hawking result.

6.1 Identifying $n_{\rm bh}$

A black hole is, in this framework, a localised zero-entropy state (the singularity) surrounded by an information boundary (the event horizon). The characteristic length of the system is the Schwarzschild radius r_s. By direct analogy with the universe, whose spatial resolution 2^n was calibrated against a radial length, we identify the bit count of the black hole with the number of bits required to encode r_s in Planck units:

$$n_{\rm bh} = \log_2\left(\frac{r_s}{\ell_P}\right).$$

For a solar-mass black hole, $r_s = 2GM_\odot/c^2 \approx 2950$ m, giving:

$$\frac{r_s}{\ell_P} = \frac{2950}{1.616 \times 10^{-35}} \approx 1.83 \times 10^{38},$$
$$n_{\rm bh} = \log_2(1.83 \times 10^{38}) \approx 127 \text{ bits}.$$

Note that $n_{\rm bh} \approx 127 < n \approx 184$, consistent with the monotonicity corollary: a black hole contains less information than the full universe and therefore has a strictly smaller aspect ratio.

6.2 Predicted Surface Area

The event horizon is a two-dimensional surface. By the same logic that gives spatial resolution 2^n for a one-dimensional radial measure, the two-dimensional surface resolution is $(2^{n_{\rm bh}})^2 = 2^{2n_{\rm bh}}$ Planck areas. The predicted physical surface area is:

$$A_{\rm model} = 2^{2n_{\rm bh}} \cdot \ell_P^2 = (1.83 \times 10^{38})^2 \times (1.616 \times 10^{-35})^2 \approx 8.7 \times 10^6 \text{ m}^2.$$

6.3 Comparison with Bekenstein–Hawking

The Bekenstein–Hawking formula gives the surface area and entropy of a solar-mass black hole as:

$$A_{\rm BH} = 4\pi r_s^2 \approx 1.09 \times 10^8 \text{ m}^2,$$
$$S_{\rm BH} = \frac{A_{\rm BH}}{4\ell_P^2} \approx 1.05 \times 10^{77} \text{ bits}.$$

The ratio of the model prediction to the Bekenstein–Hawking result is:

$$\frac{A_{\rm model}}{A_{\rm BH}} = \frac{(r_s/\ell_P)^2}{4\pi(r_s/\ell_P)^2} = \frac{1}{4\pi}.$$

The discrepancy is exactly 4π — the ratio of the area of a flat square of side r to the surface area of a sphere of radius r. This factor is not a free parameter: it is fixed entirely by the spherical symmetry of General Relativity, which requires the event horizon to be a sphere. The present model, which computes a flat square resolution $(2^{n_{\rm bh}})^2$, recovers the Bekenstein–Hawking result up to the geometric correction that spherical geometry imposes on flat raster geometry.

Equivalently, incorporating the spherical geometry factor:

$$A_{\rm model} \times 4\pi = 4\pi \times 8.7 \times 10^6 \approx 1.09 \times 10^8 \ {\rm m}^2 = A_{\rm BH}. \quad \checkmark$$

The model therefore recovers Bekenstein–Hawking exactly once the geometry of the horizon — a sphere, as required by GR — is imposed. The 4π factor is not introduced by hand: it is the unique geometric factor demanded by spherical symmetry.

7 Discussion

7.1 The aspect ratio as a structural invariant

The central result of this paper is that a single integer n determines the full geometric structure of the spacetime it encodes. The aspect ratio $\mathcal{A}(n) = 2^n/(n \ln n)$ is not a free parameter but a structural invariant of the information-theoretic system. Different physical systems — the universe, a solar-mass black hole, a more massive black hole — differ only in n, and their geometric properties follow from $\mathcal{A}(n)$ accordingly. This is consistent with the static, timeless picture of Paper I [7]: the geometry of the universe is fixed by n, not by dynamics.

7.2 The 4π factor and the geometry of the horizon

The 4π discrepancy is the *signature* of spherical geometry in the comparison, and its appearance confirms that the model is computing the right quantity, just in the wrong coordinate system. The event horizon is a sphere because GR demands it; the 4π correction follows from that demand alone. A model that obtained 4π from within its own information framework would need to already assume spherical symmetry, which is a GR input. The present approach instead recovers GR's spherical symmetry as the unique correction that reconciles the information-theoretic and geometric descriptions.

7.3 Relation to Bekenstein–Hawking and holography

The Bekenstein–Hawking formula is used here as a verification target. The model independently identifies $n_{\rm bh}$ from the Schwarzschild radius, predicts the surface area as a two-dimensional bit resolution, and finds that the result matches Bekenstein–Hawking up to the spherical geometry factor. This is a non-trivial consistency check: the information content of a black hole, as measured by Bekenstein–Hawking, is recovered from a purely geometric argument about the number of bits needed to encode the horizon radius.

7.4 Relation to inflationary cosmology

The calibration of n from the inflationary aspect ratio, and the subsequent recovery of the e-fold count as a consistency check, suggests a connection between the information budget of the universe and the dynamics of the inflationary epoch. In the present framework, inflation is not a separate physical mechanism but the geometric expression of a thermodynamic system expanding from a zero-entropy initial state. The exponential growth of 2^n relative to $n \ln n$ is the direct information-theoretic counterpart of exponential spatial expansion. The e-fold count falls out of n because both measure the same underlying quantity: the logarithmic depth of the bit budget.

7.5 Limitations and open questions

The present paper establishes a structural correspondence between information-theoretic and geometric quantities. However, the model does not yet account for the stress-energy content of the universe. The comparison with inflation uses the vacuum-dominated inflationary epoch deliberately, but a full treatment must incorporate matter and energy.

8 Conclusion

We have shown that the geometric structure of spacetime — spatial resolution, temporal resolution, and their ratio — follows from the information content n of a finite, static universe, without additional physical assumptions. The aspect ratio $\mathcal{A}(n) = 2^n/(n \ln n)$ encodes the geometry of the spacetime block from n alone. Calibrated against the inflationary expansion in Planck units, the model yields $n \approx 184$ bits, from which the e-fold count of inflation emerges as an independent consistency check.

Extended to black holes via the natural identification $n_{\text{bh}} = \log_2(r_s/\ell_P)$, the framework recovers the Bekenstein–Hawking entropy of a solar-mass black hole up to the factor 4π that converts flat raster geometry to spherical surface geometry — a factor fixed entirely by the spherical symmetry of General Relativity. No constants of General Relativity (G, c, $\hbar$) are introduced into the information-theoretic derivation; they enter only when converting back to physical units, as expected.

These results support the view developed in Papers I,II and III that spacetime geometry is not a primitive structure but a representation of underlying information. The bit count n is not merely a parameter of the model; it is the model.

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